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Show

Analysis Methods

Traditional

Background for Traditional Analysis

Decline curve analysis is a graphical procedure used for analyzing declining production rates and forecasting future performance of oil and gas wells. A curve fit of past production performance is done using certain standard curves. This curve fit is then extrapolated to predict potential future performance. Decline curve analysis is a basic tool for estimating recoverable reserves. Conventional or basic decline curve analysis can be used only when the production history is long enough that a trend can be identified.

Decline curve analysis is not grounded in fundamental theory but is based on empirical observations of production decline. Three types of decline curves have been identified; exponential, hyperbolic, and harmonic. There are theoretical equivalents to these decline curves (for example, it can be demonstrated that under certain circumstances, such as constant well backpressure, equations of fluid flow through porous media under "boundary-dominated flow" conditions are equivalent to "exponential" decline). However, decline curve analysis is fundamentally an empirical process based on historical observations of well performance. Because of its empirical nature, decline curve analysis is applied, as deemed appropriate for any particular situation, on single or multi-fluid streams. For example, in certain instances, the oil rate may exhibit an exponential decline, while in other situations it is the total liquids (oil + water) that exhibit the exponential trend. Thus, in some instances, the analysis is conducted on one fluid, sometimes on the total fluids, sometimes on the ratio (for example Water-Oil-Ratio (WOR) or even (WOR + 1)). Since there is no overwhelming justification for any single variable to follow a particular trend, the practical approach to decline curve analysis is to choose the variable (gas, oil, oil + water, WOR, WGR etc.) that results in a recognizable trend, and to use that decline curve to forecast future performance.

It is implicitly assumed, when using decline curve analysis, the factors causing the historical decline continue unchanged during the forecast period. These factors include both reservoir conditions and operating conditions. Some of the reservoir factors that affect the decline rate include; pressure depletion, number of producing wells, drive mechanism, reservoir characteristics, saturation changes, and relative permeability. Operating conditions that influence the decline rate are: separator pressure, tubing size, choke setting, workovers, compression, operating hours, and artificial lift. As long as these conditions do not change, the trend in decline can be analyzed and extrapolated to forecast future well performance. If these conditions are altered, for example through a well workover, then the decline rate determined pre-workover will not be applicable to the post-workover period.

When analyzing rate decline, two sets of curves are normally used. The flow rate is plotted against either time or cumulative production. Time is the most convenient independent variable because extrapolation of rate-time graphs can be directly used for production forecasting and economic evaluations. However, plots of rate vs. cumulative production have their own advantages; Not only do they provide a direct estimate of the ultimate recovery at a specified economic limit, but will also yield a more rigorous interpretation in situations where the production is influenced by intermittent operations.

Good engineering practice demands that, whenever possible, decline curve analysis should be reconciled with other indicators of reserves, such as volumetric calculations, material balance, and recovery factors. It should be noted that decline curve analysis results in an estimate of Recoverable Hydrocarbons, and NOT in Hydrocarbons-in-Place. Whereas the Hydrocarbons-in-Place are fixed by nature, the Recoverable hydrocarbons are affected by the operating conditions. For example a well producing from a reservoir containing 1BCF of gas-in-place may recover either 0.7 BCF or 0.9 BCF, depending on whether or not there is a compressor connected at the wellhead.

The following steps are taken for exponential decline analysis, and for predicting future flow rates and recoverable reserves:

1. Plot flow rate vs. time on a semi-log plot (y-axis is logarithmic) and flow rate vs. cumulative production on a Cartesian (arithmetic coordinate) scale.
2. Allowing for the fact that the early time data may not be linear, fit a straight line through the linear portion of the data, and determine the decline rate "D" from the slope (-*D*/2.303) of the semi-log plot, or directly from the slope (D) of the rate-cumulative production plot.
3. Extrapolate to $q = q_E$ to obtain the recoverable hydrocarbons.
4. Extrapolate to any specified time or abandonment rate to obtain a rate forecast and the cumulative recoverable hydrocarbons to that point in time.

Production Decline Equations

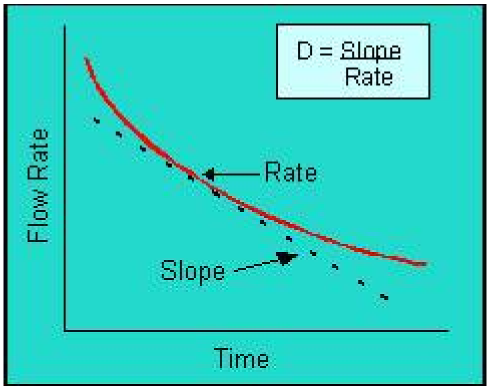
Decline curve analysis is derived from empirical observations of the production performance of oil and gas wells. Three types of decline have been observed historically: exponential, hyperbolic, and harmonic.

Decline curves represent production from the reservoir under "boundary dominated flow" conditions. This means that during the early life of a well, while it is still in "transient flow" and the reservoir boundaries have not been reached, decline curves should NOT be expected to be applicable. Typically, during transient flow, the decline rate is high, but it stabilizes once boundary dominated flow is reached. For most wells this happens within a few months of production. However, for low permeability wells (tight gas wells, in particular) transient flow conditions can last several years, and strictly speaking, should not be analyzed by decline curve methods until after they have reached stabilization.

All decline curve theory starts from the definition of the instantaneous or current decline rate (D) as follows:

$$D = -\frac{(\Delta q / q)}{\Delta t} = -\frac{(\Delta q)}{\Delta t} / q$$

D is "the fractional change in rate per unit time", frequently expressed in "% per year". In the following diagram, D = Slope/Rate.



Exponential decline occurs when the decline rate, D, is constant. If D varies, the decline is considered to be either hyperbolic or harmonic, in which case, an exponent "b" is incorporated into the equation of the decline curve, to account for the changing decline rate.

Exponential decline is given by:

$$\frac{q}{q_i} = \frac{1}{e^{Dt}}$$

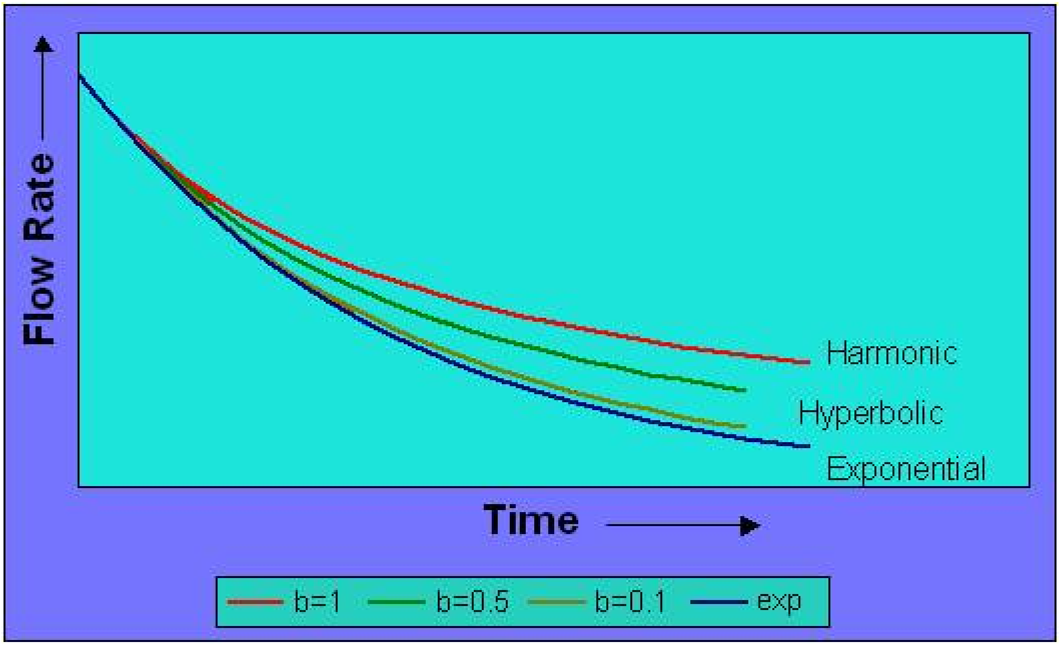
where D is the decline rate (%) and is constant.

Hyperbolic decline is described by:

$$\frac{q}{q_i} = \frac{1}{(1 + bD_i t)^{\frac{1}{b}}}$$

where D_i is the decline rate at flow rate q_i, and b is an exponent that varies from 0 to 1.

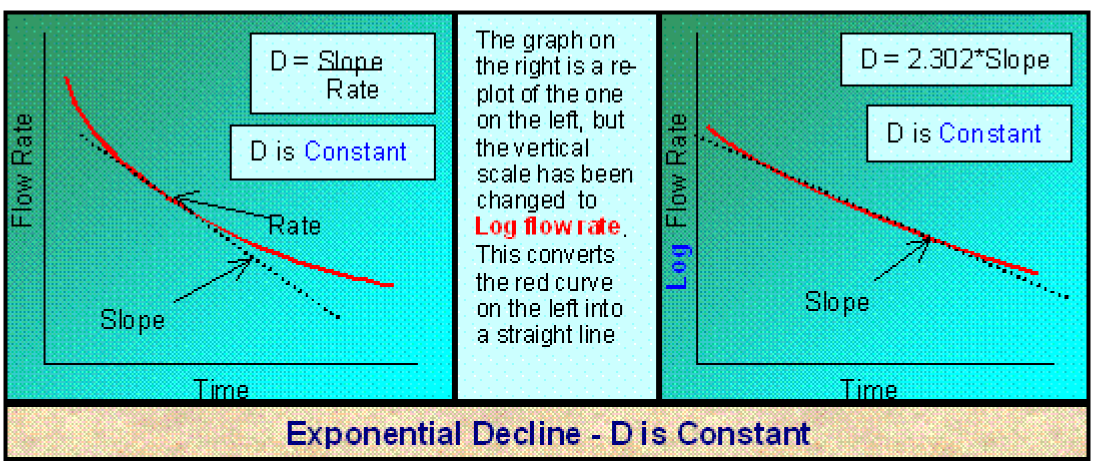
The decline rate D is not constant, so it must be associated with a specified rate, hence D_i at flow rate q_i. This is the most general form of decline equation. When b equals 1, the curve is said to be Harmonic. When 0 < b < 1, the curve is said to be Hyperbolic. When b=0, this form of the equation becomes indeterminate, however, it can be shown that it is equivalent to Exponential decline. A comparison of Exponential, Hyperbolic, and Harmonic declines is shown in the following diagram.



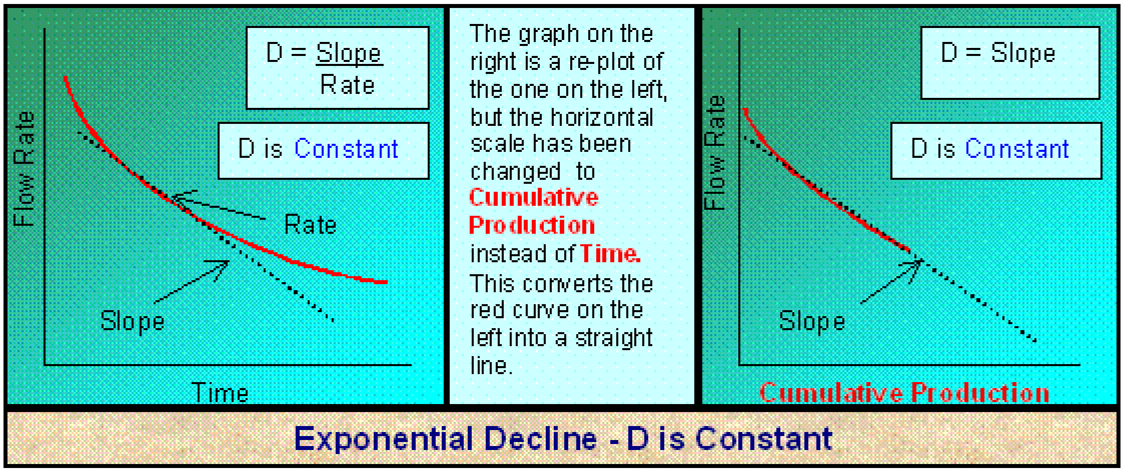
Decline curve analysis is usually conducted graphically, and in order to help in the interpretation, the equations are plotted in various combinations of "rate", "log-rate", "time," and "cumulative production". The intent is to use the combination that will result in a straight line, which then becomes easy to extrapolate for forecasting purposes. The decline equations can also be used to forecast recoverable reserves at specified abandonment rates.

Exponential Decline

When the decline rate, D, is constant, the production is said to follow an exponential decline. Another name for it is constant percentage decline. This is mathematically equivalent to a straight line on a plot of "log of production rate vs. time". Because of this simple straight-line relationship, the exponential decline is the easiest to recognize, and the simplest to use, when compared to the other decline curves (hyperbolic or harmonic). It is also the most commonly used of all the decline curves.

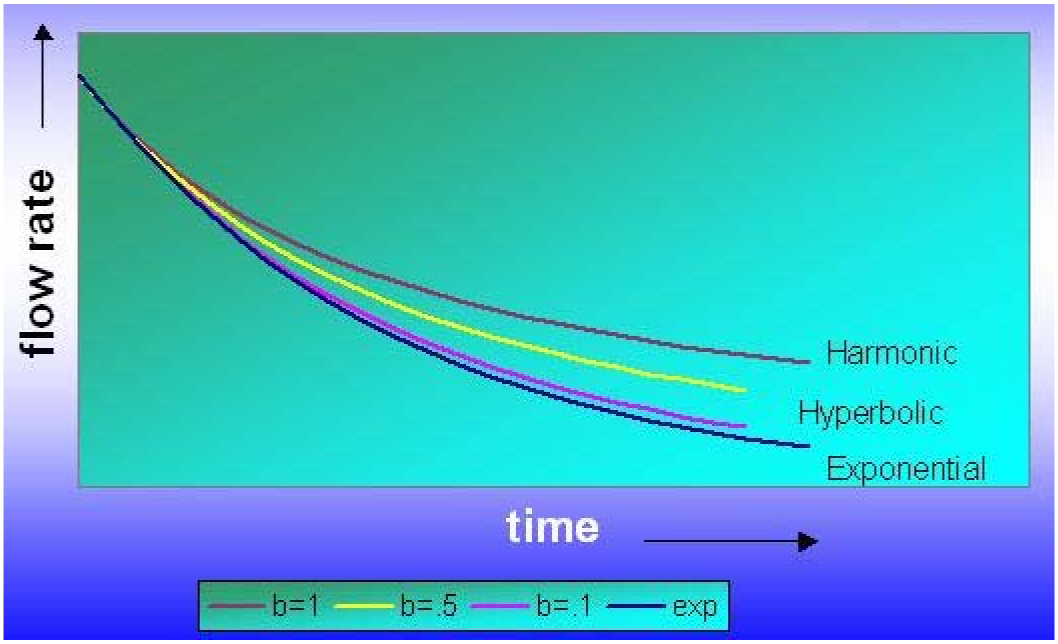


Note that, the semi-log plot of rate vs. time does not straighten ALL the rate data. This is because, initially, the well is in "transient" flow (sometimes called "flush production"). Decline curve analysis is used to model stabilized flow, not transient flow; hence, the early time data will often not follow the trend of the stabilized flow period. It can be shown that, theoretically, when a well is producing at constant backpressure, exponential decline is equivalent to "boundary dominated flow", which occurs only after the end of transient flow.



It can be shown mathematically, that an exponential decline will also result in a straight line when plotted as Flow Rate vs. Cumulative Production.

Under ideal conditions, plots of log-rate vs. time, and rate vs. cumulative production should both result in straight lines from which the decline rate can be determined. However, if the flow rate is intermittent (for example, due to market restrictions, a well only produces two weeks in any given month and is shut-in for the rest of the month), the log-rate vs. time graph will give misleading declines (because it does not account for shut-in durations), whereas the rate vs. cumulative production will give the correct straight line and decline rate. Accordingly, in general, emphasis should be placed on the rate vs. cumulative production graph, in preference to the log-rate vs. time graph.



As illustrated, exponential decline predicts a faster decline than hyperbolic or harmonic. As a result, it is often used to calculate the minimum "proved reserves".

Nominal Decline Rate

When production follows an exponential decline (also known as constant percentage decline), there are two different ways of calculating the rate forecast. They both give the same answer. One is more suited to tabular data manipulation, and uses a "nominal decline rate"; the other is the classical graphical approach based on the traditional exponential decline equation. This section attempts to show the interrelationships of the nominal and exponential decline rates.

As discussed in exponential theory the rate (q_2) at time (t_2) is related to the rate (q_1) a time (t_1) by:

$$q_2 = q_1 e^{(-D(t_2-t_1))} \qquad (1)$$

In practice, especially when rates are being calculated by hand (or calculator) and knowing that exponential decline is a constant percentage decline, some people like to calculate the rate sequence by simply multiplying the previous rate by a constant, equal to 1 minus the decline rate. (Some people refer to this procedure as constant percentage decline, but this is a misnomer because constant percentage decline is identical to exponential decline, and refers to a decline relationship and not to a method of calculation). In effect the procedure that is being applied results in the equation:

$$q_2 = q_1(1-d) \qquad (2)$$

Note the use of two different constants, "D" and "d" in Equations 1 and 2. Note also that the time interval is included in the "d" of Equation 2, but is explicit in Equation 1.

Obviously these two equations are different from each other, yet there exists a value of "d" which, when used in Equation 2, will give the same answer for q_2 as that obtained by using a different value of "D" in Equation 1, provided the time intervals between q_1 and q_2 are the same. Because the two different procedures use two different percentage declines, and yet they can produce the same rate sequence, this has caused some confusion. To clarify the situation, "d" is called the nominal decline rate, in contrast to "D" the (true) exponential decline rate. (Remember that both "d" and "D" refer to the same exponentially declining rate - also called constant percentage decline!).

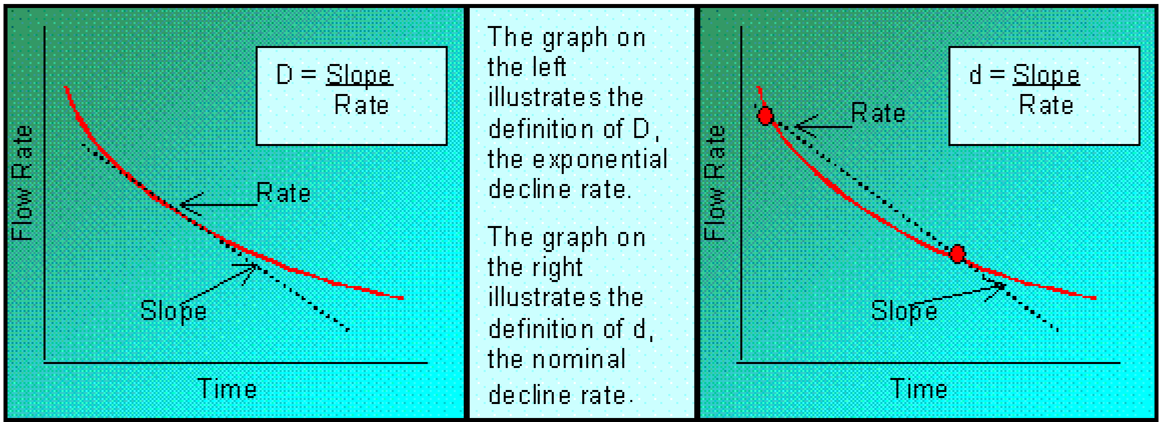
The nominal decline rate, d is defined as:

$$d = \frac{(q_1 - q_2)}{q_1}$$

The (true) exponential decline rate, D is, by definition, equivalent to:

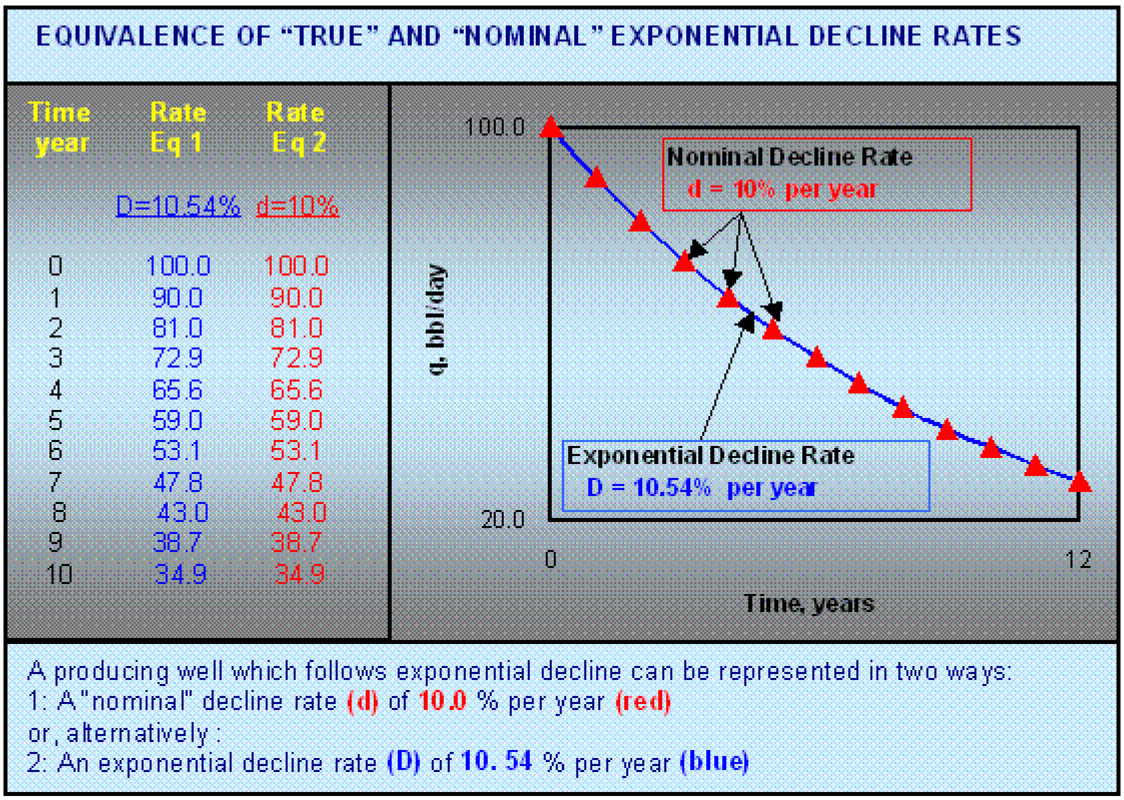
$$D = \frac{(\ln q_1 - \ln q_2)}{t_2 - t_1} = \ln\frac{(q_1/q_2)}{t_2 - t_1}$$

The difference between the two is illustrated below. In effect, "D" is related to the slope of the line, whereas "d" derives from the chord segment approximating that slope. Also, the rate by which the slope is divided is different - in the one case, it is the instantaneous rate, q; in the other case, it is the preceding rate, q_1.



Comparison of Exponential Decline Rate and Nominal Decline Rate

Usually, the nominal decline rate, d, is used when dealing with flow rate data in tabular rather than graphical format. The equivalence of the (true) exponential decline rate, D (as used in the exponential decline equation) and the "nominal" decline rate, d (as used in many hand calculations) is shown below:



The difference between d and D is usually very small, and is often within the accuracy of the data. So, even if they are misapplied, the error is usually not critical.

Decline Equations

All decline curve theory starts from the definition of the decline rate, D, as follows:

$$D = -\frac{\Delta q / q}{\Delta t}$$

For exponential decline, D is constant. Integrating the above formulation yields:

$$\ln(q/q_i) = -Dt$$

or,

$$\log(q/q_i) = -\frac{Dt}{2.303}$$

which illustrates that a plot of log-rate vs. time will yield a straight line of slope D/2.303? Cumulative Production is obtained by integrating the rate-time relationship. It can be shown that the flow rate is related to the cumulative production, Q, by:

$$q = q_i - DQ$$

which shows that a plot of rate vs. Cumulative Production will be a straight line of slope D. Extrapolation of this straight line to any specified abandonment rate (including zero) gives the recoverable reserves.

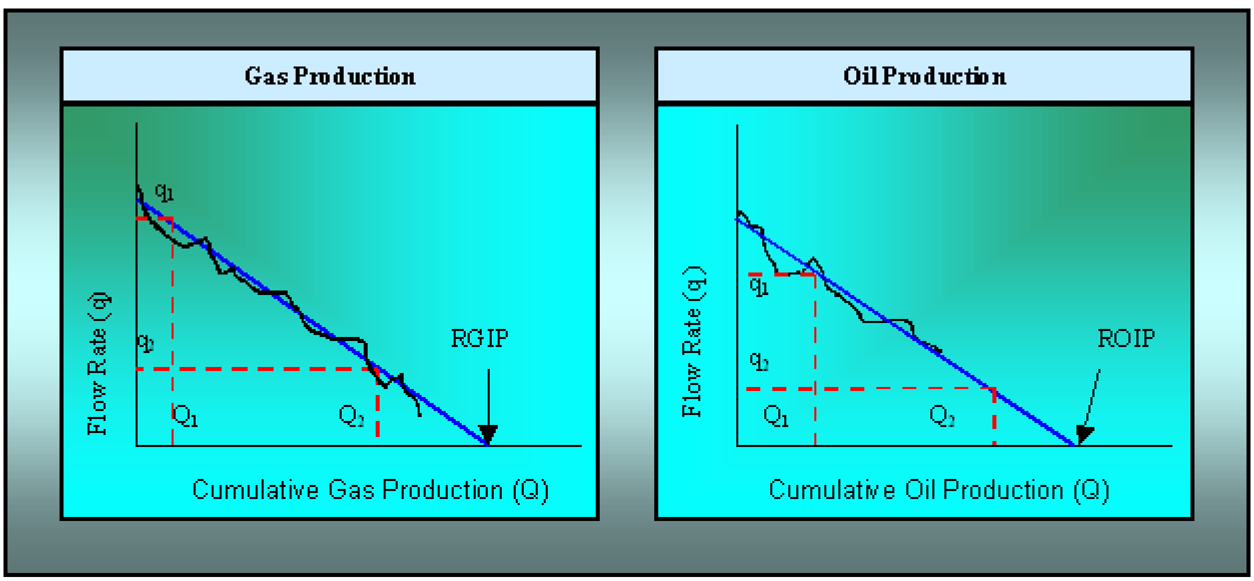
Cumulative production between time t_1 and t_2 can be obtained from

$$Q = \frac{(q_1 - q_2)}{D}$$

or, in terms of time;

$$Q = \frac{q_1}{D}\left(1 - e^{-D(t_2 - t_1)}\right)$$

The following diagram demonstrates rate-cumulative plots for oil and gas wells:



The following equation can be used for either oil or gas, provided the units are as specified below.

$$D = \frac{365(q_1 - q_2)}{1000(Q_2 - Q_1)}$$

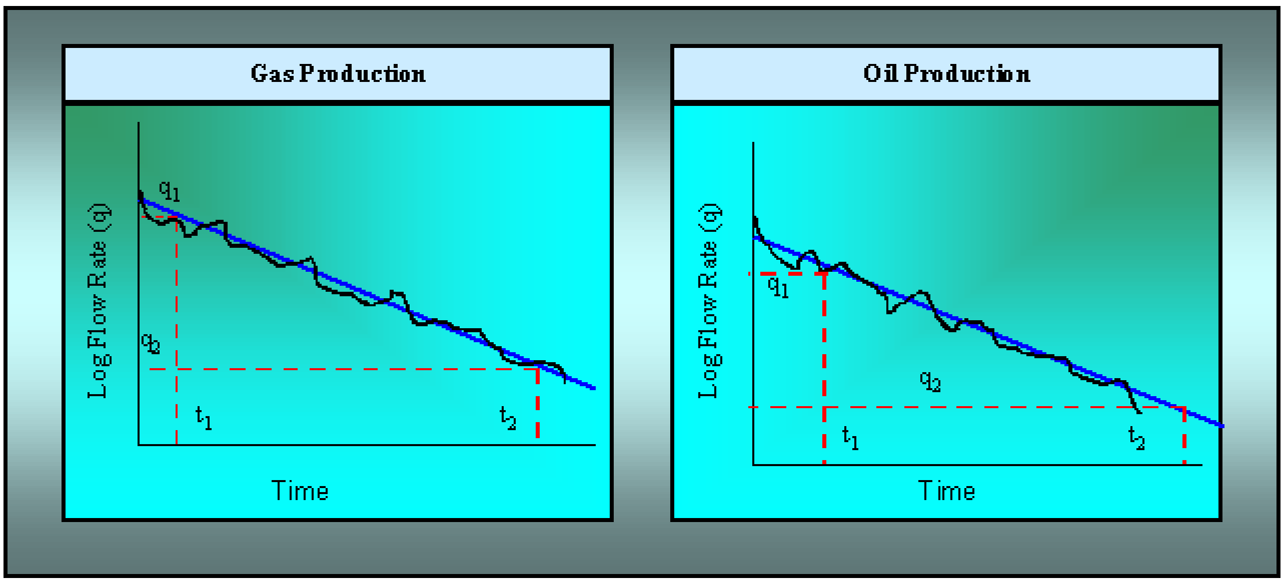
S.I. units:

for gas	for oil
$Q = 10^6\ m^3$	$Q = 10^3\ m^3$
$q = 10^3\ m^3/day$	$q = m^3/day$

Imperial units:

for gas	for oil
$Q = Bscf$	$Q = MSTB$
$q = MMscf/day$	$q = BOPD$

The following diagram demonstrates semi-log rate-time plots for oil and gas wells.



The following equation can be used for either oil or gas, provided the units are as specified below.

$$D = \frac{\ln(q_1/q_2)}{\Delta t}$$

S.I. units:

for gas	for oil

q = 10³ m³/day	*q = M* ln³*TB/day*
t = *years*	*t =* *years*

Imperial units:

for gas	**for oil**
q = MMscf/d	*q =* *BOPD*
t = *years*	*t =* *years*

Note: Time is typically measured in years. If "Time" (x-axis) on the rate-time plot is specified in days, the Dt in the decline equation is equal to: Dt (days)/365.25.

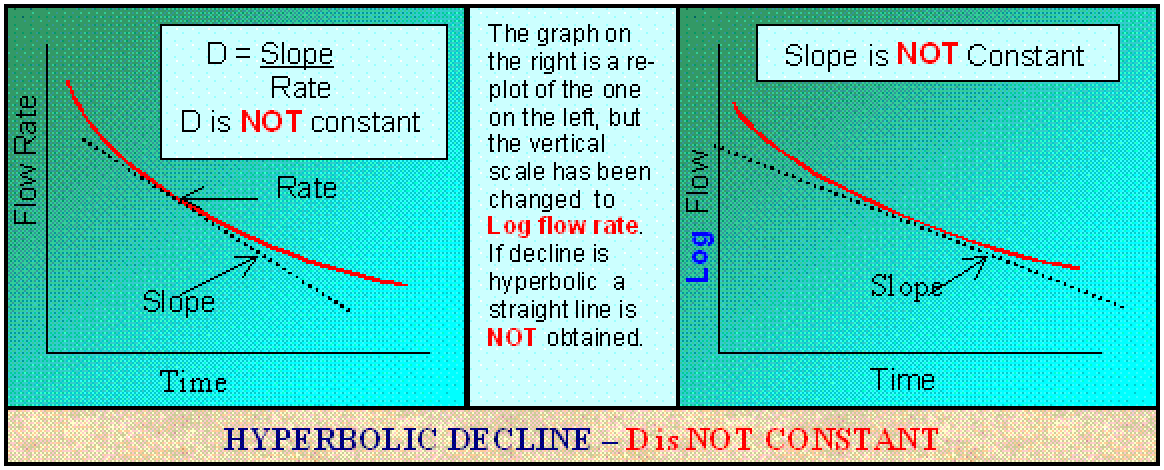
"D" from the rate-time plot (D_{RT}) should theoretically be equal to "D" from the rate-cumulative plot (D_{RC}). However, practically speaking,
$D_{RT} < D_{RC}$. If $D_{RT} > D_{RC}$ the analysis is **NOT** correct!

When forecasting remaining reserves, extrapolate the decline curve to intercept with the economic rate (q_{EC}). To calculate the RGIP or ROIP using either rate-time or rate-cumulative;

For gas, specify units Bscf or 10^6 m^3, and for oil specify units MSTB or 10^3 m^3.

Hyperbolic Decline

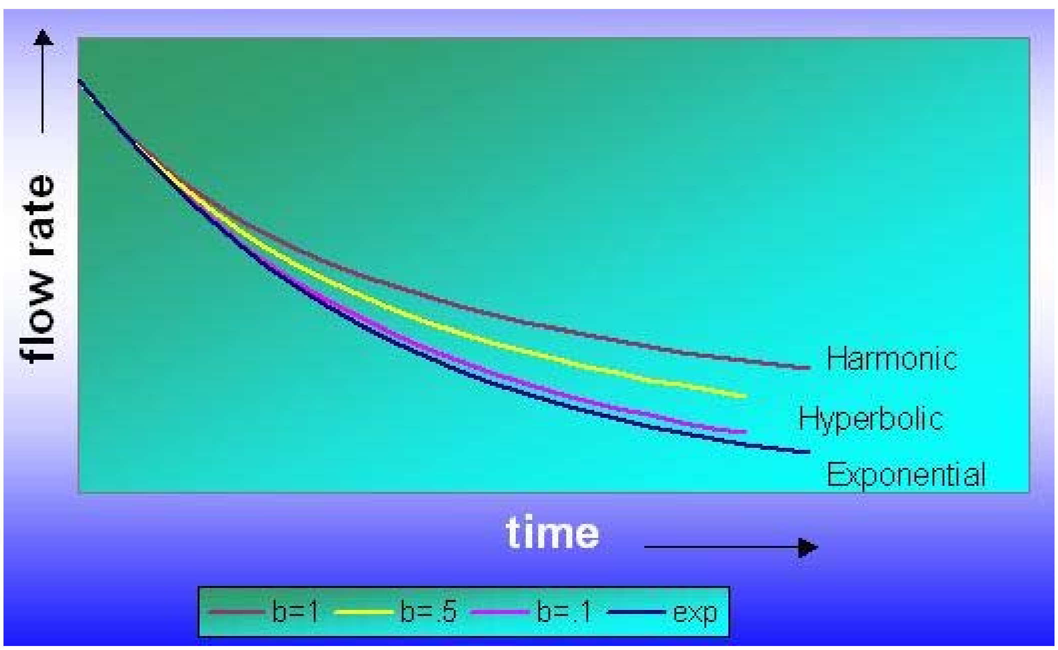
With hyperbolic decline, the decline rate, *D*, is not constant (in contrast to exponential decline, when D is constant). Empirically, it has been found that for some production profiles, D is proportional to the production rate raised to a power of *b*, where *b* is between zero and one. A value of *b* = 0 corresponds to exponential decline. A value of *b* = 1 is called harmonic decline. Sometimes, values of b > 1 are observed. These values do not conform to the traditional decline curves, and their meaning is discussed in the section entitled Definition of b.|



Unfortunately, hyperbolic decline does not plot as a linear relationship on a Cartesian grid. As shown above, plotting "Rate vs. Time" or "Rate vs. Cumulative Production" on semi-log scales does not straighten a hyperbolic decline curve either. Prior to the widespread use of personal computers, this lack of linearity was the main reason for the restricted use of hyperbolic declines.

Whereas exponential decline is consistent with our understanding of single-phase, boundary-dominated flow, hyperbolic decline is considered to be consistent with the performance of solution-gas drive reservoirs. Further discussion on this can be found under the topic definition of b.

As illustrated here, hyperbolic decline predicts a slower decline rate than exponential. As a result, it is sometimes used to calculate the "probable reserves" while exponential decline is used to obtain minimum "proved reserves".



Hyperbolic Theory

All decline curve theory starts from the definition of the decline rate, D, as follows:

$$D = -\frac{\Delta q / q}{\Delta t}$$

For hyperbolic decline, D is NOT constant and varies with the production rate according to:

$$D = Kq^b$$

where: q is the flow rate, K is a constant equal to $D_i / (q_i^b)$, and b is a constant with a value between 0 and 1. (It can be shown that the decline rate, *D,* at any time, t, is related to D_i and *b* by:

$$D = \frac{D_i}{1+bD_i t}$$

When b = 0, D becomes a constant, independent of the flow rate, q, and the hyperbolic decline becomes identical with exponential decline. When b = 1, the hyperbolic decline becomes harmonic decline.

Combining the equations above, and integrating, gives the hyperbolic decline equation:

$$q = \frac{q_i}{(1+bD_i t)^{\frac{1}{b}}}$$

where q_I *and* D_I are the initial flow rate and the initial decline corresponding to the time; t = 0, respectively.

There is no simple way of re-formulating this equation to obtain a straight line. Hence, when analyzing production data using a hyperbolic decline, a non-linear regression must be performed to determine the values of the constants *b*, D_i, and q_i that best fit the data.

In order to obtain the flow rate at any future time, the cumulative production up to that time, or the total recoverable reserves, the production decline curve must be extrapolated using the hyperbolic decline equation:

$$q = \frac{q_i}{(1+bD_i t)^{\frac{1}{b}}}$$

Having obtained the constants *b*, D_i, and q_i from a curve fit of the production data, the flow rate at any time, t, is given by:

$$q = \frac{q_i}{(1+bD_i t)^{\frac{1}{b}}}$$

The cumulative production, Q, at any time, t, is obtained is from;

$$Q_p = \frac{q_i}{(1-b)D_i}\left(1-(1+bD_i t)^{1-\frac{1}{b}}\right)$$

or in terms of *q* from:

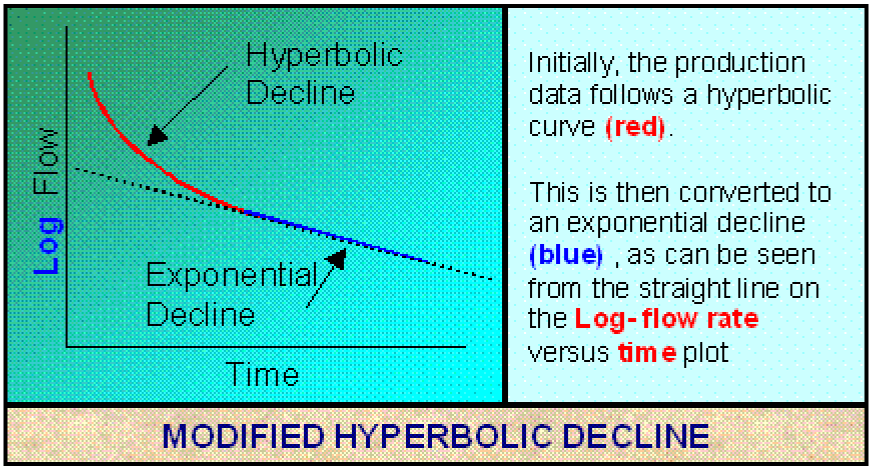
$$\Delta Q_p = \frac{q_i^b}{(1-b)D_i}\left(q_1^{1-b} - q_2^{1-b}\right) = \frac{q_i}{(1-b)D_i}\left(1-\left(\frac{q}{q_i}\right)^{1-b}\right)$$

Thus at an abandonment rate, q_2 = 0, the total recoverable oil can be read from an extrapolation of the graph based on the hyperbolic decline equation, or it can calculated from:

$$Q_{p-max} = \frac{q_i}{(1-b)D_i}$$

Modified Hyperbolic Decline

Extrapolation of hyperbolic declines over long periods of time frequently results in unrealistically high reserves. To avoid this problem, it has been suggested (Robertson) that at some point in time, the hyperbolic decline be converted into an exponential decline. Thus, assume that for a particular example, the decline rate, D, starts at 30% and decreases through time in a hyperbolic manner. When it reaches a specified value, say 10%, the hyperbolic decline can be converted to an exponential decline, and the forecast continued using the exponential decline rate of 10%.



Robertson has presented a decline equation, which incorporates the transition from hyperbolic to exponential decline:

$$q = q_i \frac{(1-\beta)^b \exp(-Dt)}{(1-\beta \exp(-Dt))^b}$$

Where *D* is the asymptotic exponential decline rate, and β *is responsible for transition from hyperbolic behaviour to exponential. Cumulative production using the above equation is given by:*

$$Q = q_i \frac{1-\beta}{\beta D}\left[1 - \frac{1}{(1-\beta \exp(-Dt))^{b-1}}\right]$$

and , for *b* = 1,

$$Q = q_i \frac{1-\beta}{\beta D} \ln\left(\frac{1-\beta \exp(-Dt)}{1-\beta}\right)$$

The difficulty with this procedure is that the choice for the value of the final exponential decline rate cannot be determined in advance, and must be specified from experience.

Alternative Modified Hyperbolic Decline Equations

The modified hyperbolic decline starts as a hyperbolic decline curve and transitions into an exponential decline curve at a specified decline rate, D, value. This value will be defined as D_{lim}. The decline rate, D, and flowrate, q, are calculated as follows:

$$D = \frac{D_i}{q_i^{\,b-1}(1+bD_i t)^{1/b}}$$ D > D_{lim}

$$q = q_i(1+bD_i t)^{-1/b}$$ D > D_{lim}

$$q = q_{lim} e^{-D_{lim}(t-t_{lim})}$$ D <= D_{lim}

where

$$q_{lim} = q_i\left(\frac{D_{lim}}{D}\right)^{1/b}$$

and

$$t_{lim} = \frac{\left(\frac{q_i}{q_{lim}}\right)^{b} - 1}{bD_i}$$

The equations can be presented as cumulative production vs. rate, also (0<b<1):

$$Q = \frac{q_i^{\,b}}{(1-b)D_i}\left(q_i^{\,1-b} - q^{1-b}\right)$$ D > D_{lim}

$$Q = \frac{q_i^{\,b}}{(1-b)D_i}\left(q_i^{\,1-b} - q_{lim}^{\,1-b}\right) + \frac{q_{lim} - q}{D_{lim}}$$ D <= D_{lim}

For b=1 *Harmonic*, the following are used instead:

$$Q = \frac{q_i}{D_i}\ln\frac{q_i}{q}$$ D > D_{lim}

$$Q = \frac{q_i}{D_i}\ln\frac{q_i}{q_{lim}} + \frac{q_{lim} - q}{D_{lim}}$$ D <= D_{lim}

To calculate EUR, the following equation is used (0<b<1):

$$EUR = Q_f + \frac{q_i^{\,b}}{(1-b)D_i}\left(q_i(1+bD_i t_f)^{\frac{b-1}{b}} - q_{lim}^{\,1-b}\right) + \frac{q_{lim} - q_{ab}}{D_{lim}}$$

where

Q_f = cumulative production at beginning of forecast period

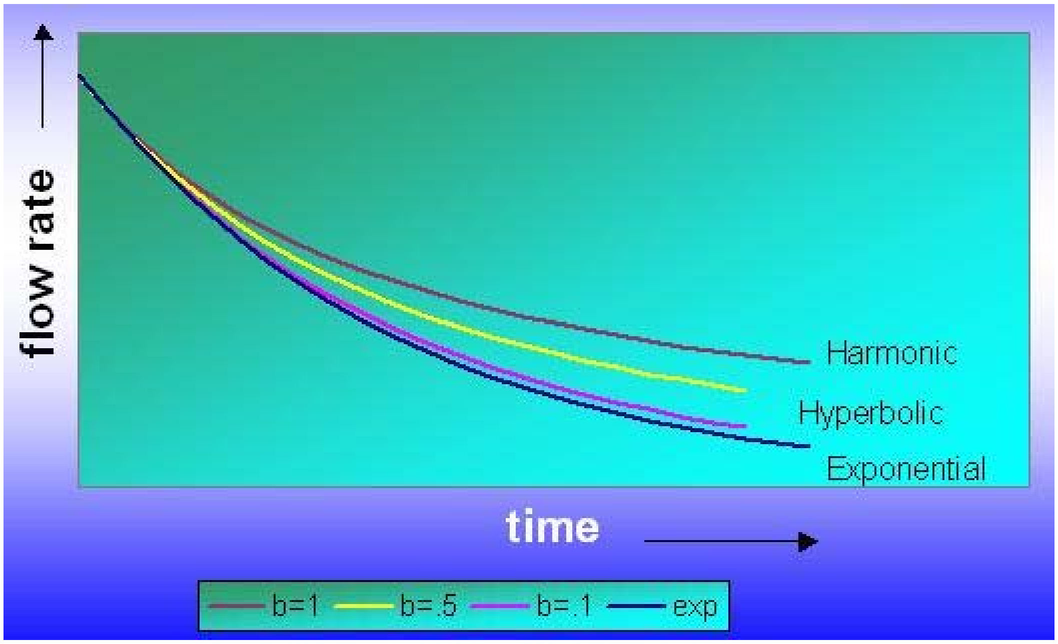
T_f = time at beginning of forecast period

For b=1 (*Harmonic)* the following is used instead:

$$EUR = Q_f + \frac{q_i}{D_i} ln\left(\frac{q_i}{q_{lim}(1 + D_i t_f)}\right) + \frac{q_{lim} - q_{ab}}{D_{lim}}$$

Harmonic Decline

Harmonic decline is a special case of hyperbolic decline, with *b* = 1, i.e., the decline rate, *D*, is proportional to *q*. This means that the decline rate, *D,* goes to zero when *q* approaches zero. This type of performance is expected when very effective recovery mechanisms such as gravity drainage are active. Another example of harmonic decline is the production of high viscosity oil driven by encroaching edge-water. Due to unfavourable mobility ratio, early water breakthrough occurs and the bulk of the oil production will be obtained at high water cuts. If the total fluid rate is kept constant then the increasing amount of water in the total fluid will cause the oil production to decline. This decline in oil rate may follow a harmonic decline.



If harmonic decline continues, the lower part of the curve will become so flat that there will be essentially no decline. This implies that the production rate will not reach zero, and thus the ultimate recoverable reserves (at zero rate) cannot be quantified, unless a (non-zero) abandonment rate is specified. Harmonic decline will become a straight line if plotted as log-rate vs. cumulative production.

Harmonic Equations

All decline curve theory starts from the definition of the decline rate, D, as follows:

$$D = -\frac{(\Delta q/q)}{\Delta t}$$

For harmonic decline, D is NOT constant; D varies with the rate according to :

$$D = K * q$$

where : K is a constant equal to D_i / q_i.

Harmonic decline is a special case of hyperbolic decline,

$$q = \frac{q_i}{(1 + bD_i t)^{\frac{1}{b}}}$$

With b = 1, this results in :

$$q = \frac{q_i}{(1 + D_i t)}$$

The cumulative production between t_1 and t_2 corresponding to the two flow rates, q_1 and q_2, can be calculated from:

$$\Delta Q_p = \frac{q_i}{D_i} ln\left(\frac{q_i}{q_2}\right)$$

or in terms of time, t, from:

$$\Delta Q_p = \frac{q_i}{D_i} ln(1 + D_i t)$$

.

The ultimate production (at zero flowrate) cannot be determined.

From the above equations, it can be seen that the way to obtain a straight line for harmonic decline is to plot log-rate vs. cumulative production. (Because harmonic decline is such a rare occurrence, this plot is not shown here). The constants D_i and q_i can be determined by regression, or from a plot of log-rate vs. cumulative production. The flow rate at any future time, the cumulative production until that time, and recoverable reserves at a specified abandonment rate can be found either from extrapolation of the curves or from the equations above.

Definition of b

Single-phase liquid production, high-pressure gas, tubing-restricted gas production, and poor waterflood performance lead to b = 0 (Fetkovich). Under solution gas drive, the lower the gas relative permeability, the smaller is the quantity of gas produced; hence the decline in reservoir pressure is slower, and accordingly the decline rate is lower (higher value of b). Simulation studies for a range of gas and oil relative permeability values have indicated $0.1 < b < 0.4$ for different k_{rg}/k_{ro} curves, with the average curve resulting in b = 0.3. Note that the production data above the bubble point should not be analysed along with data below the bubble point. As pointed out in the introduction, decline analysis is valid when the recovery mechanism and the operating conditions do not vary with time. Above the bubble point pressure, b = 0 (exponential decline), while below the bubble point b increases as discussed above for solution gas drive. Typical gas wells have b in the range of 0.4 - 0.5. Conventional (light oil) reservoirs under edge water drive (effective water drive) seem to exhibit b = 0.5 (field experience).

If there is a mechanism present that maintains reservoir pressure, the production rate would essentially remain constant (under constant producing pressure) and the decline would tend towards zero. Examples of such mechanisms could be gas or water injection, an active water drive, or gas-cap drive. Since the decline in reservoir pressure is small, the production driving force remains large, and the decline in the producing rate is correspondingly smaller. For such cases, there is no theoretical reason why the decline coefficient could not be greater than one. Much later in the life of these reservoirs, when the oil column thins, the production rate would decline exponentially and hydrocarbon production is replaced by water.

Arps' original study indicated less than 10% of wells have $b > 0.5$; however a later study by Ramsay and Guerrero indicated about 40% of leases have $b > 0.5$. Commingled layered reservoirs lead to $0.5 < b < 1.0$ (Fetkovich). It is possible, under certain production scenarios that, initially, the rate does not decline. In such a case, decline analysis should be initialized from the start of declining rate. Field experience presented by Arps (149 fields) and by Ramsay and Guerrero (202 leases) indicate $0.1 < b < 0.9$. Exponential decline seems to be a rare occurrence, yet it is probably the most common interpretation (no doubt due to the ease of analysis).

Values of b > 1

$$q = \frac{q_i}{(1 + bD_i t)^{\frac{1}{b}}}$$

where $0 < b < 1$.

A value of b = 0 corresponds to exponential decline and a value of b = 1 corresponds to harmonic decline. Values of b >1 are not consistent with decline curve theory, but they are sometimes encountered, and their meaning is explained below.

Decline curve analysis is based on empirical observations of production rate decline, and not on theoretical derivations. Attempts to explain the observed behaviour, using the theory of flow in porous media, lead to the fact that these empirically observed declines are related to "boundary dominated flow". When a well is placed on production, there will be transient flow initially. Eventually, all the reservoir boundaries will be felt, and it is only after this time, that decline curve analysis becomes applicable, and the value of "b" lies in the range of 0 to 1, depending on the reservoir boundary conditions and the recovery mechanism.

Occasionally, decline curves with values of b > 1 are encountered. Below are some reasons that have been presented to explain this:

- the interpretation is wrong, and another value of b < 1 will fit the data.
- the data is still in transient flow and has not reached "boundary-dominated flow".
- Gentry and McCray, using numerical simulation showed that reservoir layering can cause values of b > 1 (JPT 1978, 1327-41).
- Bailey showed that some fractured gas wells showed values of b > 1, (and sometimes as high as 3.5) (OGJ Feb. 15, 1982,
  117-118).

Fetkovich believes that the data have been wrongly interpreted. More on this issue is presented when the relation between recovery mechanism and b is discussed.

Calculation of EUR and Fluid in Place

Using the following, we can calculate the expected ultimate recovery for a variety of situations.

EUR Calculations

Exponential

$$EUR = Q_f + \frac{q_f - q_{ab}}{D}$$

$$q_f = q_i e^{-Dt_f}$$

where;

q_I -- initial rate. This is the starting rate of the period preceding the forecast

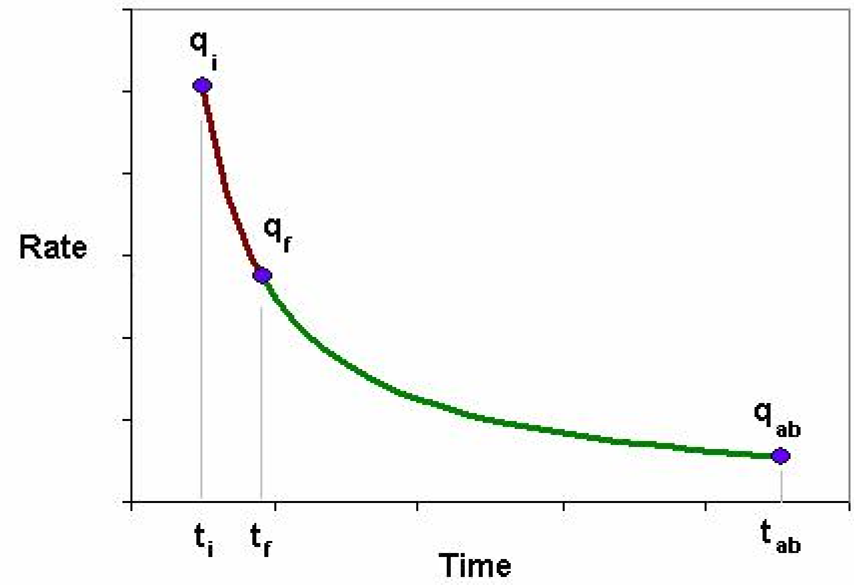
q_{ab} -- abandonment rate

q_f - rate at the beginning of the forecast period

Q_f - cumulative production at the start of the forecast

D -- decline rate

Note: The "f' subscript denotes conditions at the beginning of the forecast period.



Hyperbolic

$$EUR = Q_f + \frac{q_f}{(1-b)D_f}\left(q_i^{1-b}(1+bD_it_f)^{1-\frac{1}{b}} - q_{ab}^{1-b} \right)$$

$$q_i \rightarrow q(0)$$
$$q_f \rightarrow q(t)$$

Harmonic

$$EUR = Q_f + \frac{q_f}{D_f} ln\left(\frac{q_f}{q_{ab}}\right)$$

Fluid in Place (GIP, OIP) and Area

With knowledge of the EUR, we can estimate fluid-in-place and drainage area.

Oil

$$N = \frac{EUR}{c_t(P_i - P_{wf})}$$ (MBbls)

$$A = \frac{NB_0}{\Phi h S_0 (43560)}$$ (acres)

Gas

$$G_i = \frac{EUR \times P_i Z_{wf}}{P_i Z_{wf} - P_{wf} Z_i}$$ (bcf)

$$A = \frac{Z_i T P_{sc} G_i}{\Phi h s_g P_i T_{sc} (43560)}$$ (acres)

Table of Production Decline Equations

Production Decline Equations (Arps)			
Decline type	Constant-percentage Decline (Exponential)	Hyperbolic Decline	Harmonic Decline
Basic Characteristics	Decline is constant b=0	Decline is proportional to a fractional power (b) of the production rate 0 < b< 1	Decline is proportional to production rate b = 1
Rate-Time	$D = K * q^0 = -\frac{dq/dt}{q}$ $\int_0^t D dt = -\int_{q_i}^{q} \frac{dq}{q}$ $-Dt = \ln \frac{q}{q_i}$ $q = q_i * e^{-Dt}$	$D = K * q^b = -\frac{dq/dt}{q}$ for initial conditions $K = \frac{D_i}{q^b}$ $\int_0^t \frac{D_i}{q_i^b} * dt = -\int_{q_i}^{q} \frac{dq}{q^{b+1}}$ $\frac{bD_i t}{q_i^b} = q^{-b} - q_i^{-b}$ $q = q_i (1 + bD_i t)^{\frac{-1}{b}}$	$D = K * q^1 = -\frac{dq/dt}{q}$ for initial conditions $K = \frac{D_i}{q_i}$ $\int_0^t \frac{D_i}{q_i} dt = -\int_{q_i}^{q} \frac{dq}{q^2}$ $\frac{D_i t}{q_i} = \frac{1}{q_i} - \frac{1}{q_t}$ $q = q_i (1 + D_i t)^{-1}$

Rate - Cumulative	$Q=\int_0^t q*dt=\int_0^t q_i*e^{-Dt}*dt$ $Q=\frac{q_i-q_i*e^{-Dt}}{D}$ Substitute from Rate-time equation: $q_i*e^{-Dt}=q$ To find: $Q=\frac{qi-q}{D}$	$Q=\int_0^t q*dt=\int_0^t q_i(1+bD_it)^{\frac{-1}{b}}*dt$ $Q=\frac{q_i}{(1-b)D_i}\left[(1+bD_it)^{\frac{b-1}{b}}-1\right]$ Substitute from Rate-time equation: $(1+bD_it)=\left(\frac{q_i}{q}\right)^b$ To find: $Q=\frac{q_i^b}{(1-b)D_i}\left(q_i^{1-b}-q^{1-b}\right)$	$Q=\int_0^t q*dt=\int_0^t q_i(1+D_it)^{-1}*dt$ $Q=\frac{q_i}{D_i}\left[\ln(1+D_it)\right]$ Substitute from Rate-time equation $(1+D_it)=\frac{q_i}{q}$ To find: $Q=\frac{q_i}{D_i}\ln\frac{q_i}{q}$